Probabilistic Analysis Of A Two Non-Identical Unit Standby System With Multi-Component, Repair, Inspection And Post Repair Of A Unit.

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Abstract— The paper analyses in respect of reliability, MTSF and cost benefit a two non-identical unit cold standby system in which one of the unit is known as priority unit and the other as non-priority unit. The priority unit is made of a number of components so that functioning of each unit is necessary for its operation. The ordinary unit has only single component. After repair the priority unit goes for inspection and then it is sent for post repair if repair is not found perfect while second unit on repair works as good as new. The jobs of repair, inspection and post repair are performed by a single repairman. The analysis is performed by supplementary variable technique.

Index Terms— Inspection, MTSF, multi-component, post repair, reliability, supplementary variable, transition Probability,

1 INTRODUCTION

The study of multi-component reparable system models is of great interest for reliability engineers and industry managers due to their wide applicability. Two and three unit reparable redundant system models with single component have been already analysed by various authors [1-5] under different model formulations. They have obtained the reliability characteristics by using regenerative point technique. In past the publications [6,7] have dealt with the problem of analysing multicomponent systems for evaluating reliability, MTSF and net expected profit by using regenerative point technique.

Recently Linmin Hu et al. [8] analysed a series parallel reparable system model consisting of one master control unit, two slave units and a single repairman who operates single vacation. Using exponential failure time and arbitrary repair time distributions various steady state measures of system effectiveness have been obtained using supplementary variable technique. They have assumed single component unit. Supplementary variable technique has been popularly used to analyse cold standby systems under various assumptions [9-11].

Further most of the authors assumed in analysing the system models that repaired unit immediately goes into operation or standby as the case may be. In many real existing situations it can be observed that a repaired unit should be inspected to decide whether the repair is perfect or not. In case the repair is not perfect the unit should be sent for post repair. Also, in the context of present scenario as a unit is composed of a number of components working in series network so that failure of any component leads to complete breakdown of the unit. Keeping the above facts in view the aim of present paper is to analyse a two non-identical unit cold standby system model assuming that one unit is of complex nature consisting of a number of components connected in series and the other is a simple unit having single component. The complex unit needs inspection after each repair to decide whether the repair done is perfect or not. A single service facility is available for repair, inspection and post repair.

Being non-markov reparable systems with general distribution of many random variables the supplementary variable technique has been used to obtain integro-differential equations for the system by using simple probabilistic arguments. Further taking the laplace transforms of these equations we obtain linear first order partial differential equations solving which we get various state probabilities in terms of their laplace transform. By using these state probabilities explicit expressions for the following reliability measures have been obtained:

i). Steady state probabilities.
ii). Point wise availabilities and steady state availabilities.
iii). Reliability and MTSF.
iv). Expected up time of the system and busy periods with repair, inspection and post repair.
v). Profit gain for the system.

2 ASSUMPTIONS

i). The system consists of two non-identical units. Initially one unit is operative and the other is in cold standby. Upon failure of an operative unit the cold standby unit becomes operative instantaneously.
ii). First unit is n-component branded as well as costly unit while the other unit is a single component ordinary unit. Each unit has two modes- Normal (N) and total failure (F).
iii). The first unit being branded has lesser failure rate so priority in operation is given to the first unit while pri-

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ority in repair is given to second unit over the first unit as repair in case of second unit is less costly and takes less time.

iv). Single repairman is available with the system to repair, inspect and post repair the failed unit.

v). After repair the priority unit goes for inspection to decide whether its repair is perfect or not. If the repair is found to be perfect then the repaired unit becomes operative otherwise it is sent for post repair.

vi). The switching device is perfect and instantaneous**.**

vii). A repaired ordinary unit and post repaired priority unit works as good as new

3 Notations And States

$P_k(t)$: P [system is in state S_k at time t]; $k = 0, 1i, 2i, \ldots\ldots, 6i$

$P_k(x,t)dx$: P [system is in state S_k at time t and has sojourned in this state for duration between $(x, x+dx)$.

$*, s$: †Symbols for Laplace Transform i.e. $P_k^*(s) = L.T\left[P_k(t)\right] = \int \exp(-st) P_k(t)\,dt$

α_i : Constant failure rate of i^{th} component of first unit

θ : Constant failure rate of second unit

$g_i(x), G_i(x)$: pdf and cdf of repair time of i^{th} component of first unit so that

$$g_i(x) = \mu_i(x)\exp\left[-\int_0^x \mu_i(u)\,du\right]$$

$h_i(x), H_i(x)$: pdf and cdf of post repair of i^{th} component of first unit so that

$$h_i(x) = \eta_i(x)\exp\left[-\int_0^x \eta_i(u)\,du\right]$$

$m_i(x), M_i(x)$: pdf and cdf of inspection of i^{th} component of first unit so that

$$m_i(x) = \psi_i(x)\exp\left[-\int_0^x \psi_i(u)\,du\right]$$

$k(x), K(x)$: pdf and cdf of repair time of second unit so that

$$k(x) = \phi(x)\exp\left[-\int_0^x \phi(u)\,du\right]$$

p : probability of transition from state S_{2i} to S_0

q : probability of transition from state S_{2i} to S_{3i}

Symbol for modes

N ≡ Normal mode; F ≡Failure mode

Subscript symbols

o ≡ operative s ≡ standby

r ≡ repair pr ≡ post repair

I ≡ inspection w ≡ waiting for repair

Thus the possible modes with subscript are

N_{10} First unit in operation

N_{20} Second unit in operation

N_{2s} Second unit in standby

F_{ir} Unit in F-mode and under repair

F_{iI} Unit in F-mode and under inspection

F_{ipr} Unit in F-mode and under post repair

F_{iwr} Unit in F-mode and waiting for repair

F_{iwI} Unit in F-mode and waiting for inspection

Considering the possible modes of two units, the system may be in one of the following states:

$S_o \equiv (N_{10}, N_{2s})$, $S_{1i} \equiv (F_{ir}, N_{20})$

$S_{2i} \equiv (F_{iI}, N_{20})$, $S_{3i} \equiv (F_{ipr}, N_{20})$

$S_{4i} \equiv (F_{iwr}, F_{2r})$, $S_{5i} \equiv (F_{iwI}, F_{2r})$

$S_{6i} \equiv (F_{iwpr}, F_{2r})$

The transition rates can be displayed in matrix form:

from \ To	S_0	S_{1i}	S_{2i}	S_{3i}	S_{4i}	S_{5i}	S_{6i}
S_0	-	α_i	-	-	-	-	-
S_{1i}	-	-	$\mu_i(x)$	-	θ	-	-
S_{2i}	$p, \psi_i(x)$	-	-	$q, \psi_i(x)$	-	θ	-
S_{3i}	$\eta_i(x)$	-	-	-	-	-	θ
S_{4i}	-	$\phi(x)$	-	-	-	-	-
S_{5i}	-	-	$\phi(x)$	-	-	-	-
S_{6i}	-	-	-	$\phi(x)$	-	-	-

4 Development Of Equations And Their Solution

Having defined possible states of the system we construct the following integro-differential equations for the system by using simple probabilistic arguments:

$$\left(\frac{\partial}{\partial t} + \sum \alpha_i\right) P_0(t) = \sum \int P_{2i}(x,t)\, p\psi_i(x)\,dx + \sum \int P_{3i}(x,t)\,\eta_i(x)\,dx$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_i(x)+\theta\right)P_{1i}(x,t)=0$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\psi_i(x)+\theta\right)P_{2i}(x,t)=0$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\eta_i(x)+\theta\right)P_{3i}(x,t)=0$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi(x)\right)P_{4i}(x,t)=0$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi(x)\right)P_{5i}(x,t)=0$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi(x)\right)P_{6i}(x,t)=0 \tag{1-7}$$

Boundary conditions are

$$P_{1i}(0,t)=\alpha_i P_0(t)+\int P_{4i}(x,t)\phi(x)dx$$

$$P_{2i}(0,t)=\int P_{1i}(x,t)\mu_i(x)dx+\int P_{5i}(x,t)\phi(x)dx$$

$$P_{3i}(0,t)=\int P_{2i}(x,t)q\psi_i(x)dx+\int P_{6i}(x,t)\phi(x)dx$$

$$P_{4i}(0,t)=\theta P_{1i}(t)$$

$$P_{5i}(0,t)=\theta P_{2i}(t)$$

$$P_{6i}(0,t)=\theta P_{3i}(t) \tag{8-13}$$

Initial conditions: It is assumed that the system initially started from normal state S_0 i.e.

$$P_0(0)=1 \text{ and } P_k(x,0)=0\ ; k=1i,2i,3i,4i,5i,6i.$$

Taking Laplace Transforms of above equations (1-13) we get

$$\left(s+\sum\alpha_i\right)P_0^*(s)-\sum\int P_{2i}^*(x,s)p\psi_i(x)dx-\sum\int P_{3i}^*(x,s)\eta_i(x)dx=1$$

$$\frac{\partial}{\partial x}P_{1i}^*(x,s)+\left(s+\theta+\mu_i(x)\right)P_{1i}^*(x,s)=0$$

$$\frac{\partial}{\partial x}P_{2i}^*(x,s)+\left(s+\theta+\psi_i(x)\right)P_{2i}^*(x,s)=0$$

$$\frac{\partial}{\partial x}P_{3i}^*(x,s)+\left(s+\theta+\eta_i(x)\right)P_{3i}^*(x,s)=0$$

$$\frac{\partial}{\partial x}P_{4i}^*(x,s)+\left(s+\phi(x)\right)P_{4i}^*(x,s)=0$$

$$\frac{\partial}{\partial x}P_{5i}^*(x,s)+\left(s+\phi(x)\right)P_{5i}^*(x,s)=0$$

$$\frac{\partial}{\partial x}P_{6i}^*(x,s)+\left(s+\phi(x)\right)P_{6i}^*(x,s)=0$$

$$P_{1i}^*(0,s)=\alpha_i P_0^*(s)+\int P_{4i}^*(x,s)\phi(x)dx$$

$$P_{2i}^*(0,s)=\int P_{1i}^*(x,s)\mu_i(x)dx+\int P_{5i}^*(x,s)\phi(x)dx$$

$$P_{3i}^*(0,s)=\int P_{2i}^*(x,s)q\psi_i(x)dx+\int P_{6i}^*(x,s)\phi(x)dx$$

$$P_{4i}^*(0,s)=\theta P_{1i}^*(s)$$

$$P_{5i}^*(0,s)=\theta P_{2i}^*(s)$$

$$P_{6i}^*(0,s)=\theta P_{3i}^*(s) \tag{14-26}$$

Integrating (15) and using (21),

$$P_{1i}^*(x,s)=\exp\left[-(s+\theta)x-\int_0^x\mu_i(u)du\right]P_{1i}^*(0,s) \tag{27}$$

So that,

$$P_{1i}^*(s)=\int P_{1i}^*(x,s)dx=\int\exp\left[(-s+\theta)x-\int_0^x\mu_i(u)du\right]P_{1i}^*(0,s)dx$$

$$=\frac{\left(1-g_i^*(s+\theta)\right)}{(s+\theta)}P_{1i}^*(0,s)=A_i(s)P_{1i}^*(0,s) \tag{28}$$

Similarly,

$$P_{2i}^*(x,s)=\exp\left[-(s+\theta)x-\int_0^x\psi_i(u)du\right]P_{2i}^*(0,s) \tag{29}$$

$$P_{2i}^*(s)=\frac{\left(1-m_i^*(s+\theta)\right)}{(s+\theta)}P_{2i}^*(0,s)=B_i(s)P_{2i}^*(0,s) \tag{30}$$

$$P_{3i}^*(x,s)=\exp\left[-(s+\theta)x-\int_0^x\eta_i(u)du\right]P_{3i}^*(0,s) \tag{31}$$

$$P_{3i}^*(s)=\frac{\left(1-h_i^*(s+\theta)\right)}{(s+\theta)}P_{3i}^*(0,s)=C_i(s)P_{3i}^*(0,s) \tag{32}$$

$$P_{4i}^*(x,s)=\exp\left[-sx-\int_0^x\phi(u)du\right]P_{4i}^*(0,s) \tag{33}$$

$$P_{4i}^*(s)=\int P_{4i}^*(x,s)dx=\frac{\left(1-k^*(s)\right)}{s}P_{4i}^*(0,s)=D_i(s)P_{4i}^*(0,s) \tag{34}$$

$$P_{5i}^*(x,s)=\exp\left[-sx-\int_0^x\phi(u)du\right]P_{5i}^*(0,s) \tag{35}$$

$$P_{5i}^*(s)=\int P_{5i}^*(x,s)dx=\frac{\left(1-k^*(s)\right)}{s}P_{5i}^*(0,s)$$

$$=D_i(s)P_{5i}^*(0,s) \tag{36}$$

$$P_{6i}^*(x,s)=\exp\left[-sx-\int_0^x\phi(u)du\right]P_{6i}^*(0,s) \tag{37}$$

$$P_{6i}^*(s)=\int P_{6i}^*(x,s)dx=\frac{\left(1-k^*(s)\right)}{s}P_{6i}^*(0,s)$$

$$=D_i(s)P_{6i}^*(0,s) \tag{38}$$

Also,

$$\int P_{1i}^*(x,s)\mu_i(x)dx=\int\exp\left[-(s+\theta)x-\int_0^x\mu_i(u)du\right]\mu_i(x)P_{1i}^*(0,s)dx$$

$$=g_i^*(s+\theta)P_{1i}^*(0,s) \tag{39}$$

Similarly,

$$\int P_{2i}^*(x,s)\psi_i(x)dx=m_i^*(s+\theta)P_{2i}^*(0,s)$$

$$\int P_{3i}^*(x,s)\eta_i(x)dx=h_i^*(s+\theta)P_{3i}^*(0,s)$$

$$\int P_{4i}^*(x,s)\phi(x)dx = \theta k^*(s)P_{1i}^*(s)$$
$$\int P_{5i}^*(x,s)\phi(x)dx = \theta k^*(s)P_{2i}^*(s)$$
$$\int P_{6i}^*(x,s)\phi(x)dx = \theta k^*(s)P_{3i}^*(s) \qquad (40\text{-}44)$$

Substituting the values in (21), (22) and (23)

$$P_{1i}^*(0,s) == \alpha_i P_0^*(s) + \theta k^*(s)P_{1i}^*(s)$$
$$P_{2i}^*(0,s) = g_i^*(s+\theta)P_{1i}^*(0,s) + \theta k^*(s)P_{2i}^*(s)$$
$$P_{3i}^*(0,s) = qm_i^*(s+\theta)P_{2i}^*(0,s) + \theta k^*(s)P_{3i}^*(s) \qquad (45\text{-}47)$$

Also from (28) and (45)

$$P_{1i}^*(s) = \frac{\alpha_i A_i(s)}{\left(1-\theta k^*(s)A_i(s)\right)} P_0^*(s) \qquad (48)$$

Similarly,

$$P_{2i}^*(s) = \frac{\alpha_i B_i(s) g_i^*(s+\theta) P_0^*(s)}{\left[1-\theta k^*(s)A_i(s)\right]\left[1-\theta k^*(s)B_i(s)\right]} \qquad (49)$$

$$P_{3i}^*(s) = \frac{C_i(s) qm_i^*(s+\theta) g_i^*(s+\theta)\alpha_i P_0^*(s)}{\left[1-\theta k^*(s)A_i(s)\right]\left[1-\theta k^*(s)B_i(s)\right]\left[1-\theta k^*(s)C_i(s)\right]} \qquad (50)$$

$$P_{4i}^*(s) = \frac{D_i(s)\theta\alpha_i A_i(s) P_0^*(s)}{\left[1-\theta k^*(s)A_i(s)\right]} \qquad (51)$$

$$P_{5i}^*(s) = \frac{\theta D_i(s)\alpha_i B_i(s) g_i^*(s+\theta) P_0^*(s)}{\left[1-\theta k^*(s)A_i(s)\right]\left[1-\theta k^*(s)B_i(s)\right]} \qquad (52)$$

$$P_{6i}^*(s) = \frac{\theta D_i(s) q\alpha_i C_i(s) m_i^*(s+\theta) g_i^*(s+\theta) P_0^*(s)}{\left[1-\theta k^*(s)A_i(s)\right]\left[1-\theta k^*(s)B_i(s)\right]\left[1-\theta k^*(s)C_i(s)\right]} \qquad (53)$$

Substituting the values in (14) we get

$$P_0^*(s) = \left[\left(s+\sum\alpha_i\right) - \frac{\sum\alpha_i m_i^*(s+\theta) g_i^*(s+\theta)}{\left[1-\theta k^*(s)A_i(s)\right]\left[1-\theta k^*(s)B_i(s)\right]} \left\{p + q\frac{h_i^*(s+\theta)}{\left[1-\theta k^*(s)C_i(s)\right]}\right\}\right]^{-1} \qquad (54)$$

5 Analysis Of Characteristics

5.1 Steady State Probabilities

The steady state probabilities that the system will be in state S_0 is:

$$p_0 = \lim_{t\to\infty} P_0(t) = \lim_{s\to 0} sP_0^*(s) = \lim_{s\to 0} \frac{1}{\frac{d}{ds}\left(P_0^*(s)\right)^{-1}}$$

Let,

$$\lim_{s\to 0}\frac{d}{ds} m_i^*(s+\theta) = M_i \qquad \lim_{s\to 0}\frac{d}{ds} g_i^*(s+\theta) = G_i$$
$$\lim_{s\to 0}\frac{d}{ds} h_i^*(s+\theta) = H_i \qquad \lim_{s\to 0}\frac{d}{ds} k^*(s) = \varphi$$

Therefore,

$$\lim_{s\to 0}\frac{d}{ds} A_i(s) = -\frac{G_i}{\theta} - \frac{\left(1-g_i^*(\theta)\right)}{\theta^2}$$
$$\lim_{s\to 0}\frac{d}{ds} B_i(s) = -\frac{M_i}{\theta} - \frac{\left(1-m_i^*(\theta)\right)}{\theta^2}$$
$$\lim_{s\to 0}\frac{d}{ds} C_i(s) = -\frac{H_i}{\theta} - \frac{\left(1-h_i^*(\theta)\right)}{\theta^2}$$
$$\lim_{s\to 0}\frac{d}{ds} C_i(s) = -\frac{H_i}{\theta} - \frac{\left(1-h_i^*(\theta)\right)}{\theta^2}$$

Thus,

$$p_0 = \lim_{s\to 0}\frac{d}{ds}\left(P_0^*(s)\right)^{-1}$$
$$= \left[1 + \sum\alpha_i\left(q\frac{\left(1-h_i^*(\theta)\right)}{h_i^*(\theta)}\left(\frac{1}{\theta}-\varphi\right)\right) + \sum\alpha_i\left(\frac{\left(1-m_i^*(\theta)\right)}{m_i^*(\theta)}\left(\frac{1}{\theta}-\varphi\right)\right) + \sum\alpha_i\left(\frac{\left(1-g_i^*(\theta)\right)}{g_i^*(\theta)}\left(\frac{1}{\theta}-\varphi\right)\right)\right]^{-1}$$

$$p_{1i} = \frac{\alpha_i A_i(0)}{g_i^*(\theta)} p_0 \qquad p_{2i} = \frac{\alpha_i B_i(0)}{m_i^*(\theta)} p_0$$
$$p_{3i} = \frac{\alpha_i q C_i(0)}{h_i^*(\theta)} p_0 \qquad p_{4i} = \frac{\alpha_i\theta A_i(0) D_i(0)}{g_i^*(\theta)} p_0$$
$$p_{5i} = \frac{\alpha_i\theta B_i(0) D_i(0)}{m_i^*(\theta)} p_0 \qquad p_{6i} = q\frac{\alpha_i\theta C_i(0) D_i(0)}{h_i^*(\theta)} p_0$$

5.2 Point wise Availability

The Point-wise availability of the system in terms of its Laplace Transform is given by:

$$A^*(s) = L.T\left[A(t)\right]$$
$$= L.T\left[P_0(t) + \sum P_{1i}(t) + \sum P_{2i}(t) + \sum P_{3i}(t)\right]$$
$$= \left[1 + \sum\frac{\alpha_i A_i(s)}{\left(1-\theta k^*(s)A_i(s)\right)} + \sum\frac{\alpha_i B_i(s) g_i^*(s+\theta)}{\left[1-\theta k^*(s)A_i(s)\right]\left[1-\theta k^*(s)B_i(s)\right]} + \sum\frac{q\alpha_i C_i(s) m_i^*(s+\theta) g_i^*(s+\theta)}{\left[1-\theta k^*(s)A_i(s)\right]\left[1-\theta k^*(s)B_i(s)\right]\left[1-\theta k^*(s)C_i(s)\right]}\right] P_0^*(s)$$

5.3 Steady-state Availability

The probability that in long run system will be operative is given by:

$$A(\infty) = \lim_{s\to 0} sA^*(s) = \sum\left(p_0 + p_{1i} + p_{2i} + p_{3i}\right)$$

5.4 Reliability

The reliability of the system R (t) in terms of its Laplace Transform is $R^*(s) = L.T\left[R(t)\right]$

This can be obtained by assuming the failed states S_{4i}, S_{5i}, S_{6i} as absorbing. Thus

$$R^*(s) = \left[P_0^*(s) + \sum P_{1i}^*(s) + \sum P_{2i}^*(s) + \sum P_{3i}^*(s)\right]_{k^*(s)=0}$$

$$= \frac{\left[1 + \sum \alpha_i A_i(s) + \sum \alpha_i B_i(s) g_i^*(s+\theta) + \sum q\alpha_i C_i(s) m_i^*(s+\theta) g_i^*(s+\theta)\right]}{\left[s + \sum \alpha_i - \sum \alpha_i m_i^*(s+\theta) g_i^*(s+\theta)\left\{p + qh_i^*(s+\theta)\right\}\right]}$$

5.5 MTSF

The mean time to system failure of the system is given by

$$E(T) = \int R(t)\,dt = \lim_{s\to 0} R^*(s)$$

$$= \frac{\left[1 + \sum \alpha_i A_i(0) + \sum \alpha_i B_i(0) g_i^*(\theta) + \sum q\alpha_i C_i(0) m_i^*(\theta) g_i^*(\theta)\right]}{\left[\sum \alpha_i - \sum \alpha_i m_i^*(\theta) g_i^*(\theta)\left\{p + qh_i^*(\theta)\right\}\right]}$$

5.6 Expected up-time of the system

Let $A_i^1(t)$ and $A_i^2(t)$ be the respective probabilities that system is operative at epoch t due to operation of unit-1 or unit-2. The steady state availability of the system is given by

$$A_0^1 = \sum p_0$$

$$A_0^2 = \sum (p_{1i} + p_{2i} + p_{3i})$$

The expected up-time of the system during (0, t) due to operation of unit-1 and unit-2 are:

$$\mu_{up}^1(t) = \int_0^t A_0^1(u)\,du \quad \text{So that,} \quad \mu_{up}^{1*}(s) = \frac{A_0^{1*}(s)}{s}$$

$$\mu_{up}^2(t) = \int_0^t A_0^2(u)\,du \quad \text{So that,} \quad \mu_{up}^{2*}(s) = \frac{A_0^{2*}(s)}{s}$$

5.6 Busy period analysis

Let $B_i^{r1}(t), B_i^{I1}(t), B_i^{pr1}(t)$ and $B_i^{r2}(t)$ be the respective probabilities that repairman is busy in repair of unit-1, inspection of unit-1, post repair of unit-1 and repair of unit-2.

The steady state probabilities that repairman is busy in repair of unit-1, inspection of unit-1, post repair of unit-1 and repair of unit-2 are:

$$B_0^{r1} = \sum p_{1i} \qquad B_0^{I1} = \sum p_{2i}$$

$$B_0^{pr1} = \sum p_{3i} \qquad B_0^{r2} = \sum (p_{4i} + p_{5i} + p_{6i})$$

i). Expected busy period of repairman for repair of unit-1 during time interval (0, t) is:

$$\mu_b^{(r1)}(t) = \int_0^t B_0^{r1}(u)\,du \qquad \text{So that} \qquad \mu_b^{(r1)^*}(s) = \frac{B_0^{r1*}(s)}{s}$$

ii) Expected busy period of repairman in inspection of unit-1 during time interval (0, t) is:

$$\mu_b^{(I1)}(t) = \int_0^t B_0^{I1}(u)\,du \qquad \text{So that } \mu_b^{(I1)}(t) = \int_0^t B_0^{I1}(u)\,du$$

iii) Expected busy period of repairman in post-repair of unit-1 during time interval (0, t:) is:

$$\mu_b^{(pr1)}(t) = \int_0^t B_0^{pr1}(u)\,du \qquad \text{So that, } \mu_b^{(pr1)^*}(s) = \frac{B_0^{pr1*}(s)}{s}$$

iv) Expected busy period of the repairman for repair of unit-2 during time interval (0, t) is:

$$\mu_b^{(r2)}(t) = \int_0^t B_0^{r2}(u)\,du \qquad \text{So that, } \mu_b^{(r2)^*}(s) = \frac{B_0^{r2*}(s)}{s}$$

6 Profit Function Analysis

The net expected profit incurred in (0, t) is given by

P(t) = Total revenue in (0, t) – Expected cost in (0, t)

$$P(t) = K_0\mu_{up}^1(t) + K_1\mu_{up}^2(t) - K_2\mu_b^{(r1)}(t) - K_3\mu_b^{(I1)}(t) - K_4\mu_b^{(pr1)}(t) - K_5\mu_b^{(r2)}(t)$$

Where,

K_0 = revenue per unit of time when the system is operative due to operation of unit-1.

K_1 = revenue per unit of time when the system is operative due to operation of unit-2.

K_2 = amount paid to the repairman per unit of time when he is busy in repair of unit-1.

K_3 = amount paid to the repairman per unit of time when he is busy in inspection of unit-1.

K_4 = amount paid to repairman per unit of time when he is busy in post-repair of unit-1.

K_5 = amount paid to the repairman per unit of time when he is busy in repair of unit-2.

The expected profit per unit of time in steady state is given by

$$P = \lim_{t\to\infty}\frac{P(t)}{t} = K_0\lim_{s\to 0}s^2\mu_{up}^{1*}(s) + K_1\lim_{s\to 0}s^2\mu_{up}^{2*}(s) - K_2\lim_{s\to 0}s^2\mu_b^{r1*}(s) - K_3\lim_{s\to 0}s^2\mu_b^{I1*}(s) - K_4\lim_{s\to 0}s^2\mu_b^{pr1*}(s) - K_5\lim_{s\to 0}s^2\mu_b^{r2*}(s)$$

$$= K_0A_0^1 + K_1A_0^2 - K_2B_0^{r1} - K_3B_0^{I1} - K_4B_0^{pr1} - K_5B_0^{r2}$$

7. Numerical Results

In this section, some numerical results are illustrated for the above model in two cases:

Case I: When repair time of the component of first unit, post repair of the component of first unit, inspection of the component of first unit and repair time of second unit follows exponential distribution i.e.

$$g(x) = \mu e^{-\mu x} \qquad g^*(s+\theta) = \frac{\mu}{(s+\mu+\theta)} \qquad A(s) = \frac{1}{(s+\mu+\theta)}$$

$$h(x)=\eta e^{-\eta x} \qquad h^*(s+\theta)=\frac{\eta}{(s+\eta+\theta)} \qquad B(s)=\frac{1}{(s+\psi+\theta)}$$

$$m(x)=\psi e^{-\psi x} \qquad m^*(s+\theta)=\frac{\psi}{(s+\psi+\theta)} \qquad C(s)=\frac{1}{(s+\eta+\theta)}$$

$$k(x)=\phi e^{-\phi x} \qquad k^*(s)=\frac{\phi}{(s+\phi)} \qquad D(s)=\frac{1}{(s+\phi)}$$

$$p_0=\left[1+\left(\frac{q\alpha}{\eta}+\frac{\alpha}{\psi}+\frac{\alpha}{\mu}\right)\left(1+\frac{\theta}{\phi}\right)\right]^{-1}$$

$$A(\infty)=\left[1+\frac{q\alpha}{\eta}+\frac{\alpha}{\psi}+\frac{\alpha}{\mu}\right]\left[1+\left(\frac{q\alpha}{\eta}+\frac{\alpha}{\psi}+\frac{\alpha}{\mu}\right)\left(1+\frac{\theta}{\phi}\right)\right]^{-1}$$

$$MTSF=\frac{\left[1+\frac{\alpha}{(\mu+\theta)}+\frac{\alpha\mu}{[(\mu+\theta)(\psi+\theta)]}+\frac{q\alpha\psi\mu}{[(\mu+\theta)(\psi+\theta)(\eta+\theta)]}\right]}{\left[\alpha-\frac{\alpha\psi\mu}{[(\mu+\theta)(\psi+\theta)]}\left\{p+\frac{q\eta}{[(\eta+\theta)]}\right\}\right]}$$

$$\text{Profit}=P=K_0A_0^1+K_1A_0^2-K_2B_0^{r1}-K_3B_0^{I1}-K_4B_0^{pr1}-K_5B_0^{r2}$$
$$=K_0p_0+K_1\left[\frac{q\alpha}{\eta}+\frac{\alpha}{\psi}+\frac{\alpha}{\mu}\right]p_0-K_2\frac{\alpha}{\mu}p_0-K_3\frac{\alpha}{\psi}p_0-K_4\frac{q\alpha}{\eta}p_0$$
$$-K_5\left(\frac{\alpha}{\mu}+\frac{\alpha}{\psi}+\frac{q\alpha}{\eta}\right)\frac{\theta}{\phi}p_0$$

Case II: When repair time of the component of first unit, post repair of component of first unit, inspection of component of first unit and repair time of second unit follows lindley distribution i.e.

$$g(x)=\frac{\mu^2}{(1+\mu)}(1+x)e^{-\mu x} \qquad g^*(s+\theta)=\frac{\mu^2(s+\mu+\theta+1)}{(s+\mu+\theta)^2(1+\mu)}$$

$$A(s)=\frac{(s+\theta)(1+\mu)+\mu(2+\mu)}{(1+\mu)(s+\mu+\theta)^2}$$

$$h(x)=\frac{\eta^2}{(1+\eta)}(1+x)e^{-\eta x} \qquad h^*(s+\theta)=\frac{\eta^2(s+\eta+\theta+1)}{(s+\eta+\theta)^2(1+\eta)}$$

$$B(s)=\frac{(s+\theta)(1+\psi)+\psi(2+\psi)}{(1+\psi)(s+\psi+\theta)^2}$$

$$m(x)=\frac{\psi^2}{(1+\psi)}(1+x)e^{-\psi x} \qquad m^*(s+\theta)=\frac{\psi^2(s+\psi+\theta+1)}{(s+\psi+\theta)^2(1+\psi)}$$

$$C(s)=\frac{(s+\theta)(1+\eta)+\eta(2+\eta)}{(1+\eta)(s+\eta+\theta)^2}$$

$$k(x)=\frac{\phi^2}{(1+\phi)}(1+x)e^{-\phi x} \qquad k^*(s)=\frac{\phi^2(s+\phi+1)}{(s+\phi)^2(1+\phi)}$$

$$D(s)=\frac{(s)(1+\phi)+\phi(2+\phi)}{(1+\phi)(s+\phi)^2}$$

$$p_0=\left[1+\left(\frac{\alpha\left[\theta\mu(2+\mu)+\theta^2(1+\mu)\right]}{(\mu+\theta+1)\mu^2}+\frac{\alpha\left[\theta\psi(2+\psi)+\theta^2(1+\psi)\right]}{(\psi+\theta+1)\psi^2}+\frac{q\alpha\left[\theta\eta(2+\eta)+\theta^2(1+\eta)\right]}{(\eta+\theta+1)\eta^2}\right)\left(\frac{1}{\theta}+\frac{(2+\phi)}{\phi(\phi+1)}\right)\right]^{-1}$$

$$A(\infty)=\left[1+\frac{\frac{\alpha\left[\theta(1+\mu)+\mu(2+\mu)\right]}{(1+\mu)(\mu+\theta)^2}}{\frac{\mu^2(\mu+\theta+1)}{(\mu+\theta)^2(1+\mu)}}+\frac{\frac{\alpha\left[\theta(1+\psi)+\psi(2+\psi)\right]}{(1+\psi)(\psi+\theta)^2}}{\frac{\psi^2(\psi+\theta+1)}{(\psi+\theta)^2(1+\psi)}}+\frac{\frac{q\alpha\left[\theta(1+\eta)+\eta(2+\eta)\right]}{(1+\eta)(\eta+\theta)^2}}{\frac{\eta^2(\eta+\theta+1)}{(\eta+\theta)^2(1+\eta)}}\right]p_0$$

$$MTSF=\left[\frac{1+\left(\frac{\alpha\theta(1+\mu)+\mu(2+\mu)}{(1+\mu)(\mu+\theta)^2}\right)+\left(\frac{\alpha\theta(1+\psi)+\psi(2+\psi)}{(1+\psi)(\psi+\theta)^2}\right)\left(\frac{\mu^2(\mu+\theta+1)}{(\mu+\theta)^2(1+\mu)}\right)}{\left[\alpha-\left(\frac{\alpha\mu^2(\mu+\theta+1)}{(\mu+\theta)^2(1+\mu)}\right)\left(\frac{\psi^2(\psi+\theta+1)}{(\psi+\theta)^2(1+\psi)}\right)\left\{p+q\left(\frac{\eta^2(\eta+\theta+1)}{(\eta+\theta)^2(1+\eta)}\right)\right\}\right]}\right.$$
$$\left.\frac{\left(\frac{q\alpha\theta(1+\eta)+\eta(2+\eta)}{(1+\eta)(\eta+\theta)^2}\right)\left(\frac{\mu^2(\mu+\theta+1)}{(\mu+\theta)^2(1+\mu)}\right)\left(\frac{\psi^2(\psi+\theta+1)}{(\psi+\theta)^2(1+\psi)}\right)}{\left[\alpha-\left(\frac{\alpha\mu^2(\mu+\theta+1)}{(\mu+\theta)^2(1+\mu)}\right)\left(\frac{\psi^2(\psi+\theta+1)}{(\psi+\theta)^2(1+\psi)}\right)\left\{p+q\left(\frac{\eta^2(\eta+\theta+1)}{(\eta+\theta)^2(1+\eta)}\right)\right\}\right]}\right]$$

9 Conclusion

In this paper we successfully obtained some reliability measures of the system such as the steady state probabilities, point wise availabilities and steady state availabilities, Reliability and MTSF, expected up time of the system and busy periods with repair, inspection and post repair, profit gain for the system. From a concrete study of system behaviour we observe in case1 the behaviour of MTSF w.r.t inspection rate ψ for varying values of α and μ when $\eta=\mathbf{1.5}, \theta=\mathbf{0.8}, \phi=\mathbf{1.5}$. The curve in fig. 1 depicts that MTSF increases very slow with inspection rate. Further we also conclude that the increase in MTSF with the increase of inspection rate becomes fast for less values of failure rate α. It can also be seen that MTSF decreases with increase in failure rate α and increases with increase in repair rate μ.

Fig 2 depicts that variation in steady state profit w.r.t inspection rate ψ for varying values of α and ϕ when $\eta=\mathbf{1.5}, \theta=\mathbf{0.8}, \mu=\mathbf{0.5}, K_0=\mathbf{200}, K_1=\mathbf{400}, K_2=\mathbf{350}, K_3=\mathbf{250}, K_4=\mathbf{300}, K_5=\mathbf{200}$ We may clearly observe the trend that profit gradually in-

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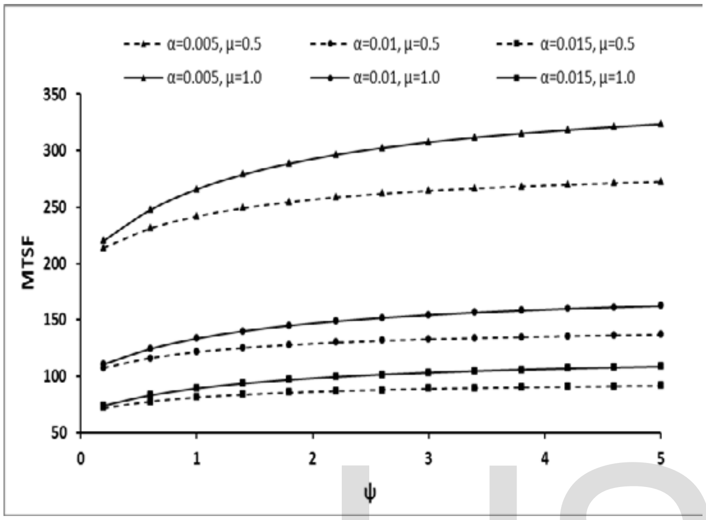
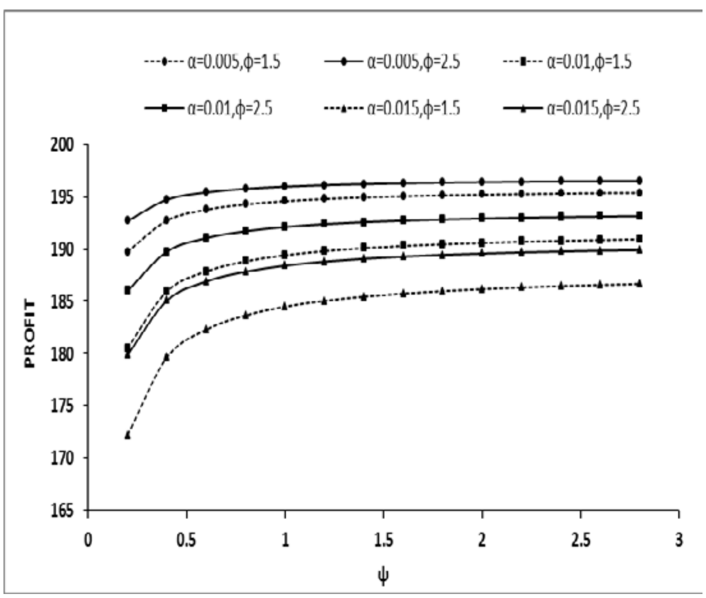
creases with inspection rate ψ and decreases with increasing failure rate α and increases with the increase in repair rate μ. Similar trends of MTSF and profit were observed in case 2 considering Lindley distribution.

10. Figures

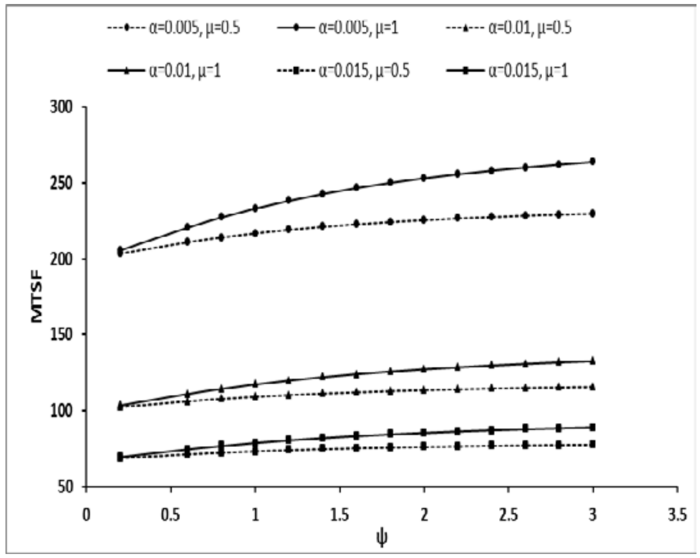
Case I: BEHAVIOUR OF MTSF W.R.T. ψ FOR VARYING VALUES OF α AND μ when $\eta = 1.5, \theta = 0.8, \phi = 1.5$



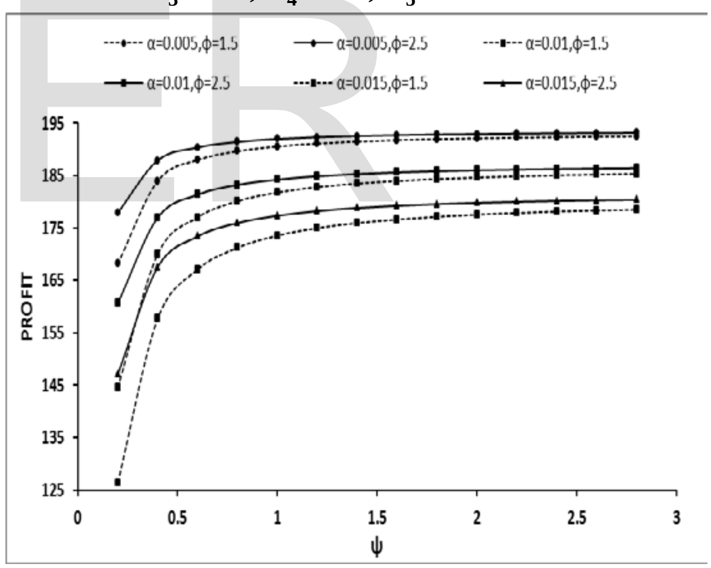
BEHAVIOUR OF PROFIT W.R.T. ψ FOR VARYING VALUES OF α AND ϕ when $\eta = 1.5, \theta = 0.8, \mu = 0.5, K_0 = 200, K_1 = 400, K_2 = 350, K_3 = 250, K_4 = 300, K_5 = 200$



Case II: BEHAVIOUR OF MTSF W.R.T. ψ FOR VARYING VALUES OF α AND μ when $\eta = 1.5, \theta = 0.8, \phi = 1.5$



BEHAVIOUR OF PROFIT W.R.T. ψ FOR VARYING VALUES OF α AND ϕ when $\eta = 1.5,\ \theta = 0.8,\ \mu = 0.5,\ K_0 = 200,\ K_1 = 400,\ K_2 = 350,\ K_3 = 250,\ K_4 = 300,\ K_5 = 200$



References

1. L.R. Goel , R.K. Agnihotry, and R. Gupta, "Stochastic analysis of a two-unit warm standby system with fault detection and inspection", Microelectron Reliab., Vol.-30, No. 1, pp. 61-65 (1990).

ISSN 2229-5518

2. L.R. Goel, R. Gupta, and R.K. Agnihotry, "Analysis of a three-unit redundant system with two types of repair and inspection", Microelectron Reliab., Vol.-29, No. 5, pp. 769-773 (1989).
3. L.R. Goel, R. Gupta and S.K Singh, "Cost analysis of a two unit warm standby system with inspection, repair and post repair", IEEE, Trans. Reliab. R-35 (1) (1986).
4. R. Gupta, C.P. Bajaj, and S.K Singh, "Cost-benefit analysis of a single server three unit redundant system with inspection, delayed replacement and two types of repair", Microelectron Reliab., Vol.-26, No. 2, pp. 247-253 (1986).
5. R. Gupta, V. Tyagi and P.K Tyagi, "Cost benefit analysis of a two unit standby system with post repair, activation time and correlated failure and repair", J. of Quality in Maintenance Engineering, 3 (1) pp. 55-63 (1997).
6. R. Gupta, C.P. Bajaj and S.M. Sinha, "A single server multi-component two unit cold standby system with inspection and imperfect switching device, Microelectron. Reliab. 26 (5) 873-877 (1986).
7. R. Gupta, C.P. Bajaj and S.M Sinha, "Cost benefit analysis of a multi-component standby system with inspection and slow switch", Microelectron. Reliab. 26 (5) 879-882 (1986).
8. L. Hu, D. Yue and J. Li, "Probabilistic analysis of a series parallel repairable system with three units and vacation", Applied Mathematical Modelling, 34 () 2711-2721 (2010)..
9. K.N.F. Leung, Y.L Zhang, and K.K Lai, "Analysis for a two-dissimilar-component cold standby repairable system with repair priority", Reliability Engineering & System Safety, 96 (11) 1542–1551 (2011).
10. H.X. Wang and G.Q. Xu, "A cold system with two different components and a single vacation of the repairman", Applied Mathematics and Computation, 219 (5) 2634–2657 (2012).
11. W.Z. Yuan and G.Q Xu, "Modelling of a deteriorating system with repair satisfying general distribution", Applied Mathematics and Computation, 218 (11) 6340–6350 (2012).